Name solutions

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EE 311 Final Exam Fall 2008

December 19, 2008

Closed Text and Notes

1) Be sure you have 18 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) Write neatly, if your writing is illegible then print.

4) The last 6 pages contain equations that may be of use to you.

5) This exam is worth 200 points.

(5 pts) 1. In spherical coordinates, a unit normal vector to the plane $\phi = 45^o$ is

A) $\mathbf{a}_r$

B) $\mathbf{a}_\theta$

[✓ C) $\mathbf{a}_\phi$]

D) none of the above

(6 pts) 2. Convert the point (2, 45°, 45°) in spherical coordinates to rectangular coordinates.

$$x = r \sin\theta \cos\phi = 2 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} = 1$$

$$y = r \sin\theta \sin\phi = 2 \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} = 1$$

$$z = r \cos\theta = 2 \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$(1, 1, \sqrt{2})$$

(6 pts) 3. The intersection of the surfaces defined by $\rho = 10$ and $\phi = \pi/4$ is a

a) a point

[✓ b) a straight line]

c) a circle

d) a plane

e) a cone

d) a cylinder

(6 pts) 4. In free space a plane of charge $17.708\text{x}10^{-12}$ C/m^2 is located at z = -1 m. The electric field intensity at the origin is,

a) $8.854\text{x}10^{-12}\hat{a}_z \frac{V}{m}$

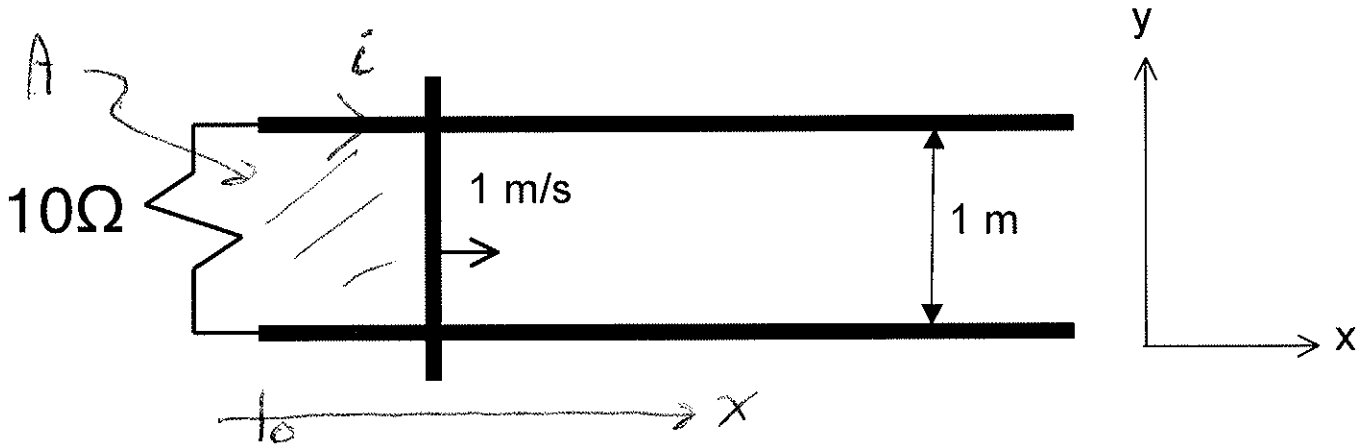
b) $-8.854\text{x}10^{-12}\hat{a}_z \frac{V}{m}$

[✓ c) $1\hat{a}_z \frac{V}{m}$]

d) $-1\hat{a}_z \frac{V}{m}$

e) none of the above

(8 pts) 5. The sliding rod in the following figure is being pulled in the +x direction as shown at 1 m/s on two rails that are separated by 1m. Everywhere is a magnetic flux density of $10\hat{a}_z T$



(5 pts) What is the current flowing through the resistor?

$$A = x(1m)$$

$$\phi = BA = (10T)(1m)(x) = \left(10\frac{Wb}{m^2}\right)(1m)(x)$$

$$\phi = \left(10\frac{Wb}{m}\right)(x)$$

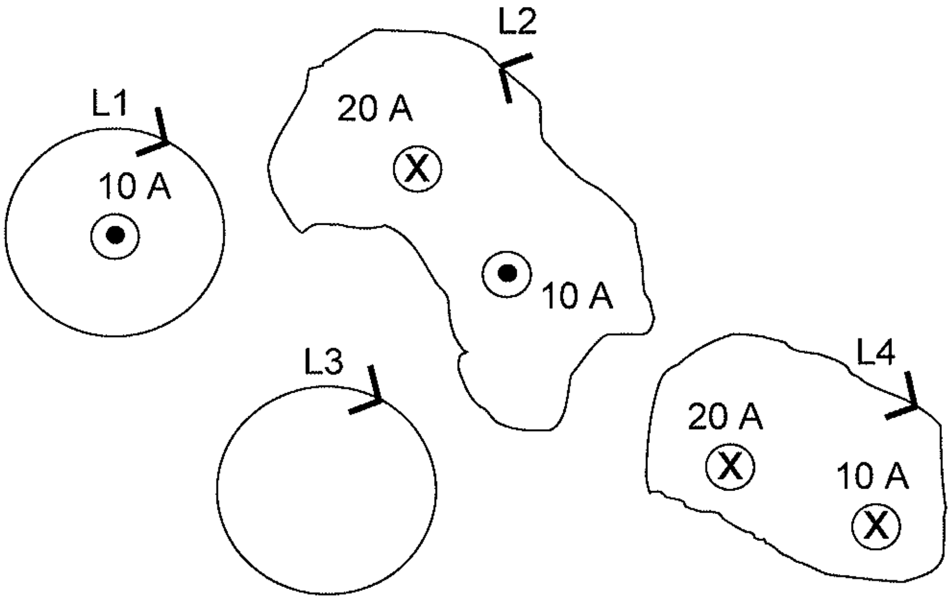
$$\frac{d\phi}{dt} = \left(10\frac{Wb}{m}\right)\left(\frac{dx}{dt}\right) = \left(10\frac{Wb}{m}\right)\left(1\frac{m}{s}\right) = 10\frac{Wb}{s}$$

$$\frac{d\phi}{dt} = 10\frac{Wb}{s} = 10\frac{Vs}{s} = 10V = |V_{emf}|$$

$$i = \frac{10V}{10\Omega} = 1A$$

(3 pts) With an arrow on the figure indicate the direction the current is flowing.

(12 pts) 6. For the currents and closed paths shown, calculate the value of the indicated integrals.



$\oint_1 \mathbf{H} \bullet \mathbf{dl} = -10A$

$\oint_2 \mathbf{H} \bullet \mathbf{dl} = -10A$

$\oint_3 \mathbf{H} \bullet \mathbf{dl} = 0\ A$

$\oint_4 \mathbf{H} \bullet \mathbf{dl} = 30\ A$

(6 pts) 7. Suppose a uniform electric field exists in the room in which you are taking this exam such that the electric field lines are horizontal and at right angles to one wall. As you walk toward the wall from which the electric field lines emerge into the room, (so if you were a positive charge you would be pushing against the force due to the electric field) are you walking toward

a) points of the same potential (along an equipotential line).
b) points of lower potential.
[✓ c)] points of higher potential.

(6 pts) 8. A parallel-plate capacitor connected to a battery stores twice as much charge with a given dielectric as it does with air as dielectric. The susceptibility of the dielectric is

A) 0
[✓ B)] 1
C) 2
D) 3
E) 4

$C_1 = \epsilon_r \epsilon_0 \frac{A}{d}$

$C_2 = \epsilon_0 \frac{A}{d}$

$Q_1 = C_1 V_1 = 2Q_2 = C_2 V$

$C_1 = 2C_2$

$\epsilon_r \epsilon_0 \frac{A}{d} = 2\epsilon_0 \frac{A}{d} \Rightarrow \epsilon_r = 2$

$\epsilon_r = 1 + \chi \Rightarrow \chi = 1$

(10 pts) 9. The region for z < 0 is free space and for z > 0 a dielectric of relative dielectric constant 2. There is a sheet of charge on the xy plane of $1\frac{C}{m^2}$. If the electric field for z < 0 is,

$$\mathbf{E} = \frac{2}{8.854x10^{-12}}\mathbf{a}_x + \frac{1}{8.854x10^{-12}}\mathbf{a}_y + \frac{7}{8.854x10^{-12}}\mathbf{a}_z\frac{V}{m}$$, then the electric field for z > 0 is,

[✓ a)] $\mathbf{E} = \frac{2}{8.854x10^{-12}}\mathbf{a}_x + \frac{1}{8.854x10^{-12}}\mathbf{a}_y + \frac{4}{8.854x10^{-12}}\mathbf{a}_z\frac{V}{m}$

b) $\mathbf{E} = \frac{2}{8.854x10^{-12}}\mathbf{a}_x + \frac{1}{8.854x10^{-12}}\mathbf{a}_y + \frac{5}{8.854x10^{-12}}\mathbf{a}_z\frac{V}{m}$

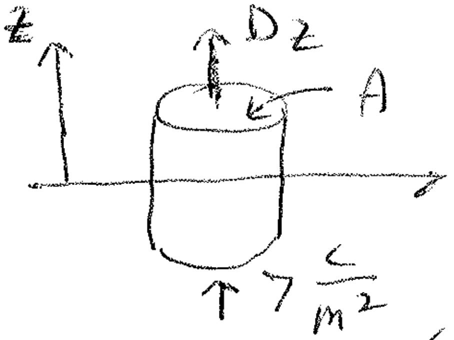
c) $\mathbf{E} = \frac{2}{8.854x10^{-12}}\mathbf{a}_x + \frac{1}{8.854x10^{-12}}\mathbf{a}_y + \frac{3}{8.854x10^{-12}}\mathbf{a}_z\frac{V}{m}$

d) $\mathbf{E} = \frac{3}{8.854x10^{-12}}\mathbf{a}_x + \frac{2}{8.854x10^{-12}}\mathbf{a}_y + \frac{5}{8.854x10^{-12}}\mathbf{a}_z\frac{V}{m}$

e) $\mathbf{E} = \frac{1}{8.854x10^{-12}}\mathbf{a}_x + \frac{0}{8.854x10^{-12}}\mathbf{a}_y + \frac{3}{8.854x10^{-12}}\mathbf{a}_z\frac{V}{m}$

f) none of the above

The tangential components of the E-field (x+y) are the same

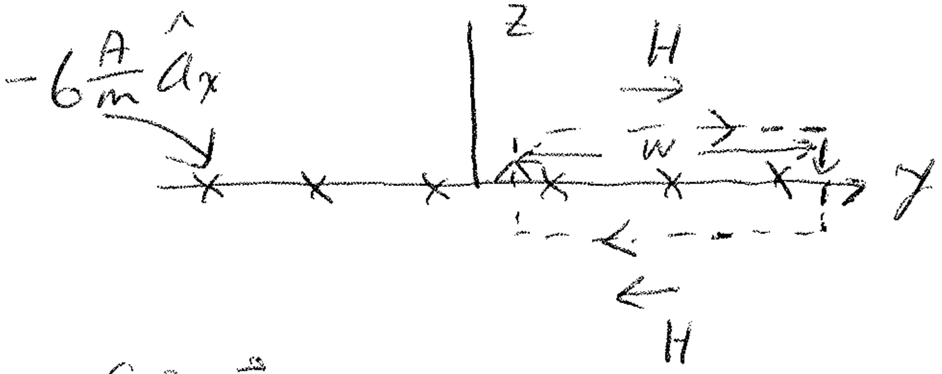


$$D_z A - (7\tfrac{C}{m^2})A = (1\tfrac{C}{m^2})A$$

$$D_z = 8\frac{C}{m^2}$$

$$E_z = \frac{D_z}{2\epsilon_0} = \frac{4}{\epsilon_0}$$

(10 pts) 10. Flowing in the xy plane is a sheet current density of $-6\hat{a}_x\frac{A}{m}$. Derive the magnetic field intensity everywhere. (Hint use Ampere's circuit law.)



$$\oint \vec{H}\cdot d\vec{l} = I_{encl} = (+6\tfrac{A}{m})w$$

$$Hw + Hw = 6w$$

$$2H = 6\frac{A}{m}$$

$$H = 3\frac{A}{m}$$

$$\vec{H} = 3\frac{A}{m}\hat{a}_y \quad z>0$$

$$= -3\frac{A}{m}\hat{a}_y \quad z<0$$

(10 pts) 11. How much energy is stored in a collection of three +1C charges that are all in a line and with 1 m between them as shown?

$$W = \frac{1}{2}\sum Q_k V_k \qquad V_1 = V_3 = \frac{1C}{4\pi\epsilon_0(1m)} + \frac{1C}{4\pi\epsilon_0(2m)}$$

$$= \frac{3}{8\pi\epsilon_0}\frac{C}{m}$$

$$V_2 = 2\frac{1C}{4\pi\epsilon_0(1m)} = \frac{4}{8\pi\epsilon_0}\frac{C}{m}$$

$$W = \frac{1}{2}\left[(1C)\frac{3}{8\pi\epsilon_0}\frac{C}{m} + (1C)\frac{4}{8\pi\epsilon_0}\frac{C}{m} + (1C)\frac{3}{8\pi\epsilon_0}\frac{C}{m}\right]$$

$$W = \frac{1}{2}\frac{10}{8\pi\epsilon_0}\frac{C^2}{m} = \frac{5}{8\pi(8.854\times10^{-12}\frac{F}{m})}\frac{C^2}{m}$$

$$W = 2.25\times10^{10}\frac{C^2}{F} = 2.25\times10^{10}\ J$$

$$\frac{C^2}{F} = \frac{C^2V}{C} = CV = C\frac{J}{C} = J$$

(10 pts) 12. The region for z < 0 is free space and for z > 0 a non-magnetic material with relative dielectric constant 2. There is a sheet current density of $-1\hat{a}_x\frac{A}{m}$ flowing on the xy plane. If the magnetic field intensity for z < 0 is,

$\mathbf{H} = 2\mathbf{a}_x + 1\mathbf{a}_y + 4\mathbf{a}_z\frac{A}{m}$, the magnetic field intensity for z > 0 is,

a) $\mathbf{H} = 2\mathbf{a}_x + 1\mathbf{a}_y + 4\mathbf{a}_z\frac{A}{m}$

b) $\mathbf{H} = 2\mathbf{a}_x + 1\mathbf{a}_y + 5\mathbf{a}_z\frac{A}{m}$

c) $\mathbf{H} = 2\mathbf{a}_x + 1\mathbf{a}_y + 3\mathbf{a}_z\frac{A}{m}$

d) $\mathbf{H} = 2\mathbf{a}_x + 0\mathbf{a}_y + 2\mathbf{a}_z\frac{A}{m}$

e) $\mathbf{H} = 1\mathbf{a}_x + 1\mathbf{a}_y + 5\mathbf{a}_z\frac{A}{m}$

[✓ f) none of the above]

(15 pts) 13. The region of space for z < 0 is filled with a dielectric of dielectric constant $\varepsilon_1 = 2\varepsilon_o$ and permeability μ_o. The region for z > 0 is filled with a dielectric of dielectric constant ε_2 and permeability μ_o. The electric field for z < 0 is given by

$$\mathbf{E}(x,t) = 30\cos[(2.51\times10^{15})t - (1.26\times10^{7})z]\mathbf{a}_x - 10\cos[(2.51\times10^{15})t + (1.26\times10^{7})z]\mathbf{a}_x \ \frac{V}{m}$$

(5 pts) What is the standing wave ratio?

$$S = \frac{E_{max}}{E_{min}} = \frac{40}{20} = 2$$

(5 pts) What is the reflection coefficient?

$$\Gamma = \frac{-10}{30} = -\frac{1}{3}$$

(5 pts) Give an expression for the magnetic field for z < 0.

$$H(r,t) = \frac{30}{\eta_1}\cos[(2.51\times10^{15})t - (1.26\times10^{7})z]\,\hat{a}_y$$

$$+ \frac{10}{\eta_1}\cos[(2.51\times10^{15})t + (1.26\times10^{7})z]\,\hat{a}_y$$

(20 pts) 14. Answer the following question for an electromagnetic field that has the following magnetic field intensity that is described by $\mathbf{H}(x,t) = 30\cos[(1.26\times10^{15})t + (1.26\times10^{7})x]\mathbf{a}_z \ \frac{V}{m}$

(5 pts) The frequency of the wave is

[✓ A)] $2x10^{14}\,Hz$

B) $1.26\times10^{7}\,Hz$

C) $2\times10^{6}\,Hz$

D) $1.26x10^{15}\,Hz$

$\omega = 1.26\times10^{15} = 2\pi f$

$f = 2\times10^{14}$

(5 pts) The wave number of the wave is

A) $1.26\times10^{15}s^{-1}$

[✓ B)] $1.26\times10^{7}\mathrm{m}^{-1}$

C) 1×10^{8}

D) none of the above

(5 pts) The velocity of the wave is

A) $1\times10^{8}\frac{m}{s}\hat{a}_z$

B) $1\times10^{8}\frac{m}{s}\hat{a}_z$

C) $1\times10^{8}\frac{m}{s}\hat{a}_x$

[✓ D)] $-1\times10^{8}\frac{m}{s}\hat{a}_x$

D) $-3\times10^{8}\frac{m}{s}\hat{a}_x$

$u = \frac{\omega}{k} = \frac{1.26\times10^{15}}{1.26\times10^{7}}$

(5 pts) The wavelength of the wave is

A) $4.99\times10^{-15}m$

[✓ B)] $4.99\times10^{-7}\mathrm{m}^{-1}$

C) $1.26\times10^{7}m$

D) $1.26\times10^{15}m$

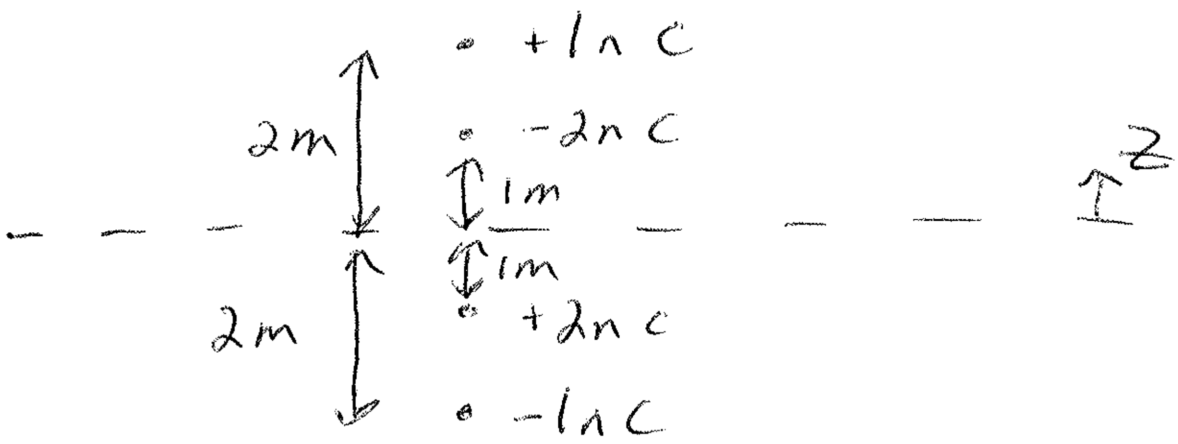
$k = \frac{2\pi}{\lambda}$

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.26\times10^{7}} = 4.99\times10^{-7}$

(10 pts) 15. Two point chare of -2 nC and 1 nC are placed, respectively, at (0, 0, 1m) and (0, 0, 2m) while an infinite conducting plane is at z = 0.

(2 pts) How much charge is induced on the plane? + 1 nC

(8 pts) What is the force on the 1 nC charge?



$$\vec{F} = \frac{1}{4\pi\epsilon_0}\left[\frac{(1nC)(-2nC)}{(1m)^2} + \frac{(1nC)(2nC)}{(3m)^2} + \frac{(1nC)(-1nC)}{(4m)^2}\right]\hat{a}_z$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{-2\times10^{-18}\,C^2}{1m^2} + \frac{2\times10^{-18}\,C^2}{9m^2} - \frac{1\times10^{-18}\,C^2}{16m^2}\right]\hat{a}_z$$

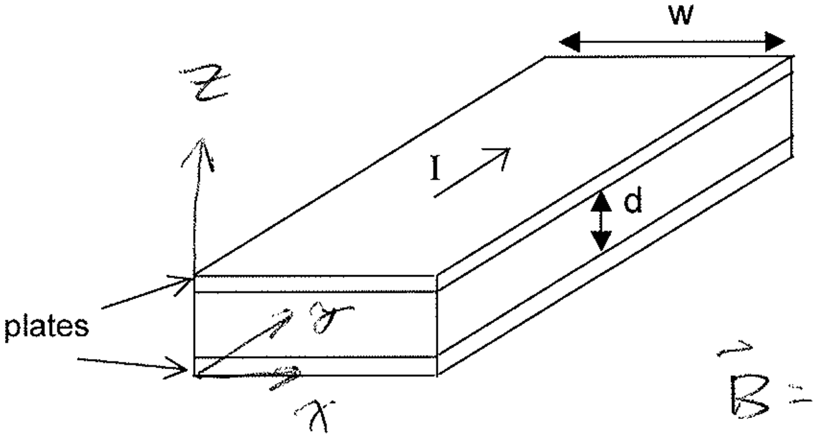
$$= \frac{1}{4\pi(8.854\times10^{-12}\,\frac{F}{m})}\left[-1.84\times10^{-18}\,\frac{C^2}{m^2}\right]\hat{a}_z$$

$$= -1.65\times10^{-8}\,N\,\hat{a}_z$$

$$\frac{C^2}{Fm} = \frac{C^2}{\frac{C}{V}m} = \frac{CV}{m}$$

$$\frac{CV}{m} = \frac{CJ}{mC} = \frac{J}{m} = \frac{Nm}{m} = N$$

(15 pts) 16. Consider a structure consisting of two plates of width w and separation d. Assuming w >> d, determine the inductance per unit length. Hint the condition w >> d allows you to determine the magnetic field between the plates as if the plates were infinite in extend. Also, if the current in the top plate is I as shown, the current flowing in the bottom plate is –I.



$$L = \frac{\psi}{I}$$

$$\vec{H} = -\frac{I}{w}\hat{a}_x, \quad 0<z<d$$

$$\vec{B} = \mu_0\vec{H} = -\frac{\mu_0 I}{w}\hat{a}_x$$

$$\psi = \iint \vec{B}\cdot d\vec{s} = \int_0^d\int_0^l \left(-\frac{\mu_0 I}{w}\hat{a}_x\right)\cdot dy\,dz\,\hat{a}_x$$

$$\psi = \frac{\mu_0 I}{w}d\,l$$

$$L = \frac{\psi}{I} = \frac{\mu_0 d\,l}{w}$$

$$\frac{L}{l} = \frac{\mu_0 d}{w}$$

(45 pts) 17. Indicate whether the following statements are true or false.

In an orthogonal coordinate system, the coordinates are mutually perpendicular.	[✓ True]	False
The magnetic flux density is a scalar	True	[✓ False]
Electromagnetic waves travel faster in dielectrics than in free space	True	[✓ False]
The frequency of an electromagnetic wave remains the same as it travels from free space into a dielectric.	[✓ True]	False
In a linear dielectric the polarization field varies linearly with the electric field intensity.	[✓ True]	False
The electric field intensity for an equipotential object is always tangential to the surface.	True	[✓ False]
The dielectric strength is the maximum electric field that a dielectric can tolerate or withstand without electric breakdown.	[✓ True]	False
In a lossy dielectric the electromagnetic wave loses power because of reflection of the wave.	True	[✓ False]
The displacement current density is the time rate of change of the electric flux density.	[✓ True]	False
The magnetic flux density is measured in T or Wb/m^2	[✓ True]	False
If the current is doubled flowing through a wire coil, the inductance of the coil is doubled.	True	[✓ False]
Two waves of equal magnitude, but propagating in opposite directions, will result in a standing wave.	[✓ True]	False
A polarization field cannot result from an applied electric field in a material made up of non-polar atoms or molecules.	True	[✓ False]
In a ferromagnetic material like iron, the individual atoms have large permanent magnetic dipole moments.	[✓ True]	False
The skin depth is independent of the frequency of the electromagnetic wave.	True	[✓ False]